

## Optical response of an anisotropic superlattice as a negative phase velocity medium

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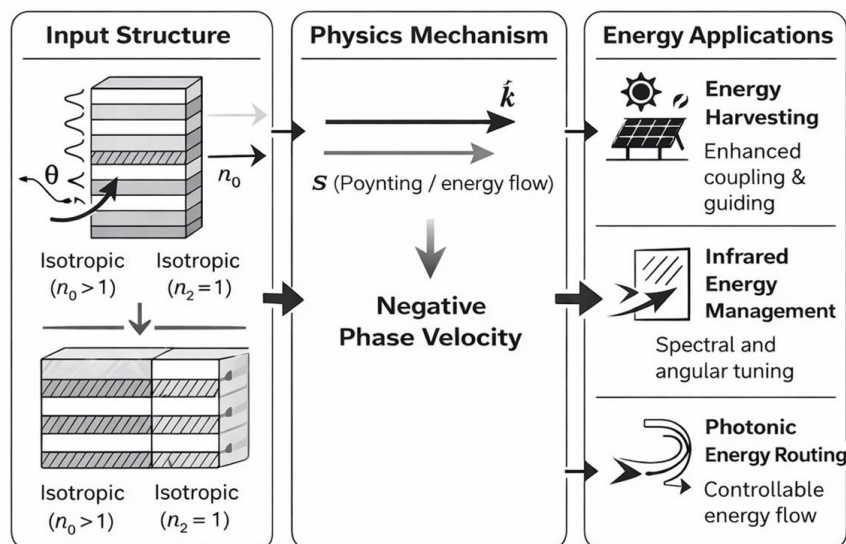
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### Abstract

Precise control of electromagnetic energy propagation is a fundamental requirement for next-generation energy-related photonic technologies. In this work, a non-magnetic anisotropic superlattice is investigated as an effective platform for realizing negative phase velocity and engineered energy transport at infrared frequencies. The structure consists of periodically repeated anisotropic thin films embedded between two isotropic dielectric environments with refractive indices  $n_0$  and  $n_2$ . By appropriate tuning of the incident angle and geometric parameters, negative phase velocity propagation is achieved without magnetic activity, enabling unconventional control over electromagnetic energy flow. The optical response of the superlattice is analyzed using a transfer-matrix formalism combined with Bloch theory for p-polarized electromagnetic waves. The combined influence of layer thickness, incident angle, and operating frequency on reflectivity spectra and Bloch wave dispersion is systematically examined. The results identify tunable spectral regimes in which phase propagation is decoupled from energy transport, leading to left-handed behavior and highly controllable electromagnetic energy transmission. From an application-oriented perspective, the proposed anisotropic superlattice offers a versatile route toward compact photonic components for energy harvesting, infrared energy management, and electromagnetic energy routing. The ability to tailor energy flow through structural and angular tuning makes this platform attractive for integration into energy-efficient waveguides and advanced optical architectures for energy-processing technologies.

### Graphical Abstract



**Anisotropic superlattice enabling tunable negative phase velocity and engineered electromagnetic energy transport**

## 1. Introduction

Materials with negative refractive index or meta-materials in which the electrical permittivity and magnetic permeability are simultaneously negative have been the subject of attractive research [1-4]. Theoretical prediction of the possibility of materials with negative refractive index was made in 1968 [5]. It was proved that materials with negative electric permittivity and magnetic permeability can be obtained without contradicting Maxwell's equations. In 2000, it was shown that it was possible to make synthetic materials with a negative refractive index [6]. In these materials, the propagation vector is oriented in the opposite direction to the Poynting vector. Although it has been suggested to make such materials in the range of visible light [6, 7], but so far these materials have been made in the microwave field [8]. The consequence of this proposal was the optical response of the sandwich array consisting of an anisotropic thin layer between two isotropic layers [9]. In one of the existing reports, it is suggested to make a highly non-anisotropic non-magnetic material with negative phase velocity [10]. Optical transmission spectra for s-polarized (TE) and p-polarized (TM) waves in one-dimensional photonic crystals on a quasi-periodic multilayer structure made by alternating SiO<sub>2</sub> and metamaterial layers, organized by following the Octonacci sequence, was reported [11]. The effect of temperature on surface waves with negative phase velocity is presented in a numerical study [12]. In a constant volume fraction of semiconductor InSb in isotropic partnering material, the angular range for negative phase velocity propagation varies considerably with temperature [12]. As reported, Dirac semi-metals, like metamaterials, could have a negative refractive index. In these materials, the collision of photons with a special dispersion relationship stimulates the resonance of electrons in Dirac cones. It is equivalent to the situation where we reach a certain resonant frequency in metamaterials from below [13]. Homogeneous materials that have a negative refractive index in the high frequency range have been investigated [14]. The TM-TE hybrid electromagnetic wave analysis, which is propagated in various multilayer anisotropic metasurfaces and in different directions, is performed using a 4×4 generalized T-matrix formalism, and the desired linear optical response is calculated [15]. Since homogeneous optical materials have significant advantages over existing heterogeneous metamaterials (i.e., isotropy, ease of construction, low losses), this approach can lead to significant improvements in materials with a negative index [14]. In this work, we intend to investigate the optical response of a superlattice consisting of multiple layers of a structure similar to the mentioned sandwich structure. In the simulation, the p-polarization of EM is considered.

## 2. Theory

Suppose we have a thin layer  $d_1$  thick and the dielectric response tensor  $\vec{\epsilon}$  sandwiched between two regions with refractive indices  $n_0$  and  $n_2$ . The thickness of each of these areas is considered  $d_0$  and  $d_2$ , respectively. On the left is the thin layer of the area with a refractive index of  $n_0$  and on the right of that area with a refractive index of  $n_2$ . The coordinate system is selected so that the xy plane is parallel to the interface and the xz plane contains the propagation plane. In isotropic environments, the relation  $k_{zi}^2 + Q^2 = n_i^2(\frac{\omega^2}{c^2})$  is established, where  $i = 0, 2$ ,  $Q$  is the component of the wave vector parallel to the interface, and  $k_{zi}$  is the component of the wave vector perpendicular to the interface. The anisotropic thin film is capable of passing two different modes of electromagnetic waves: the ordinary wave and the extraordinary wave.

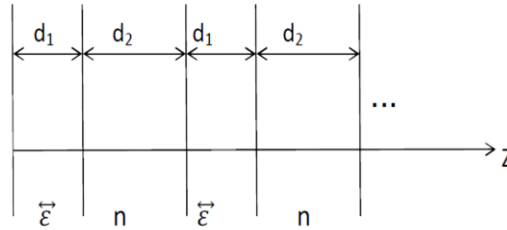
The ordinary wave is not affected by  $\varepsilon_{\parallel}$  and  $\varepsilon_{\perp}$ , while they affect the extraordinary wave. We now consider the propagation of an extraordinary wave on the  $xz$  plane. The dispersion relation as shown in [8] is  $k_{z1}^2 = \varepsilon_{\perp} v k^2$ , where  $v = 1 - [\frac{Q^2}{\varepsilon_{\parallel} k^2}]$ ,  $\varepsilon_{\parallel}$  and  $\varepsilon_{\perp}$  the elements of  $\vec{\varepsilon}$  and  $k = \omega / c$ .

Considering  $k_{z1} = k \sqrt{v \varepsilon_{\perp}}$  in the anisotropic region,  $k_{z1}$  depends on the signs of  $v$  and  $\varepsilon_{\perp}$ . If one of them is negative, this component will become an imaginary wave vector and we will not have a wave. If both are positive, we will have a wave propagation and if both are negative, the wave vector will have a negative value, and as a result, we will have a wave propagation with a negative phase velocity. This issue has been investigated in [10]. In this work, we intend to expand the mentioned report and examine the superlattice consisting of repeating the mentioned arrangement.

To use the parameters in references [8, 10], it is assumed that the anisotropic region consists of a composition of ten percent *SiC* nanospheroids, with an aspect ratio of  $1/2$ , among ninety percent quartz so that the nanoparticles are arranged in such a way that the shorter axis of all of them are in the direction of the  $x$ -axis. In this case, for  $\text{CO}_2$  laser irradiation, we will have a wavelength of  $12 \mu\text{m}$ :  $\varepsilon_{\perp} \cong -2.7 + i 1.6 \times 10^{-4}$  and  $\varepsilon_{\parallel} \cong 1.6 + i \times 10^{-5}$  [8].

To make it easier to investigate the problem, we use dimensionlessness based on the radiation frequency  $\omega_0$  ( $\lambda_0 = 12 \mu\text{m}$ ):  $\omega/\omega_0 \rightarrow \Omega$ ,  $d\omega_0/c \rightarrow D$  and  $K_{zi} = k_{zi} c / \omega_0 \rightarrow k_{zi}$ , where  $d = d_1 + d_2$  ( $d_1$  and  $d_2$  are defined in **Fig. 1**).

**Fig. 1** shows a schematic of the intended superlattice. The propagation of the electromagnetic wave in the positive direction of the  $x$ -axis and the incident medium on the superlattice with a refractive index of  $n_0$  are assumed greater than one.



**Fig. 1.** Superlattice schema.

On the other hand, using the following equation (surface impedance formula), and the reflectivity can be achieved [8]:

$$R = |r|^2 = \left| \frac{Z_v - Z_p(0)}{Z_v + Z_p(0)} \right|^2 \quad (1)$$

Where,  $r$  is the reflection coefficient,  $z_v = (n_0 k / k_{z1})$  is surface impedance of the wave incident medium on the superlattice and  $Z_p(0) = [E_x(0)/H_y(0)]$  is the surface impedance of the film.

Using Maxwell equations and applying the boundary conditions in the anisotropic region, we have:

$$B_y = \frac{\varepsilon_{\parallel} k_{z1}^2 + Q^2 \varepsilon_{\perp}}{\left(\frac{\omega}{c}\right) \varepsilon_{\parallel} k_{z1}} E_x \quad (2)$$

which can be briefly represented by the relation  $B_y = \alpha E_x$ . The transfer matrices of each isotropic and anisotropic region can be obtained as follows (for the isotropic region, parameter  $A$  is used, which is obtained from **Eq. (2)** assuming  $\varepsilon_{\parallel} = \varepsilon_{\perp} = \varepsilon$ ):

$$M_U = \begin{bmatrix} \cos(k_{z1} d_1) & \frac{i}{\alpha} \sin(k_{z1} d_1) \\ i\alpha \sin(k_{z1} d_1) & \cos(k_{z1} d_1) \end{bmatrix} \quad (3)$$

$$M_I = \begin{bmatrix} \cos(k_{z2} d_2) & \frac{i}{A} \sin(k_{z2} d_2) \\ iA \sin(k_{z2} d_2) & \cos(k_{z2} d_2) \end{bmatrix} \quad (4)$$

For each repeating element of the superlattice, the transfer matrix can be written as follows:

$$M = M_I M_{II} \quad (5)$$

Assuming  $d = d_1 + d_2$  and using Bloch's theory for periodic structure, we will have:

$$\begin{bmatrix} E_x \\ B_y \end{bmatrix}_{z+d} = e^{ipd} \begin{bmatrix} E_x \\ B_y \end{bmatrix}_z \quad (6)$$

Where  $p$  is the one-dimensional Bloch wave vector. To calculate from **Eq. (6)** we obtain:

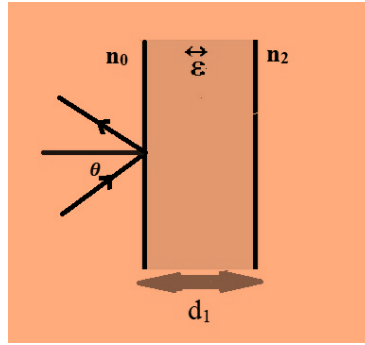
$$e^{ipd} = \frac{1}{2}[M_{11} + M_{22}] \pm \sqrt{\left(\frac{1}{4}\right)(M_{11} + M_{22})^2 - 1} \quad (7)$$

Given the loss in the environment and the direction of wave propagation, the correct sign of the relation  $|e^{ipd}| < 1$ , it will be obtained. The reflectivity can also be obtained from the following relation [16]:

$$R = |r|^2 = \left| \frac{\cos \theta / n_0 - Z_{p(0)}}{\cos \theta / n_0 + Z_{p(0)}} \right|^2 \quad (8)$$

where  $\theta$  is the incident angle. We also have a surface impedance [16]:

$$Z_p(0) = -\frac{M_{12}}{M_{11} - e^{ipd}} = -\frac{M_{22} - e^{ipd}}{M_{21}} \quad (9)$$

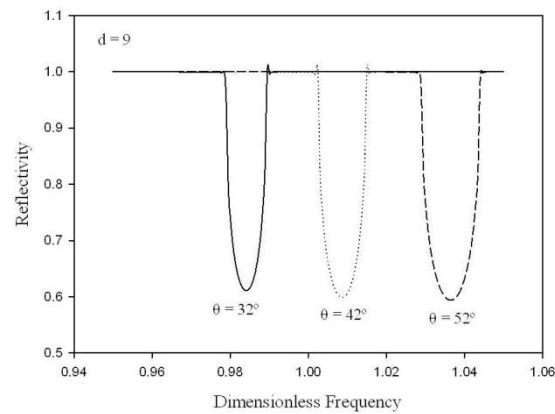


**Fig. 2.** Symbolic representation of the system under study consisting of a heterogeneous and anisotropic layer with a thickness of  $d_1$  between two layers with refractive indices of  $n_0$  and  $n_2$ .

The geometry considered is shown in **Fig.2**. A medium with a refractive index of  $n_0=2$  is considered a coupling prism. Irradiation of light on the film at an angle  $\theta$ , where  $\theta_{c0} < \theta$  and  $\theta_{c0}$  satisfies the condition  $\varepsilon_{\parallel} < n_0^2 \sin^2 \theta_{c0}$ , can cause electromagnetic propagation with a negative phase velocity in the anisotropic film. In this work, a prism with refractive index  $n_0=2$ , an anisotropic layer with thickness  $d_1$  with the following elements of the dielectric tensor  $\varepsilon_{\perp} \cong 2.7i1.6 \times 10^{-4}$ ,  $\varepsilon_{\parallel} \cong 1.6 + i \times 10^{-5}$  and  $n_2=1$  are considered. Use  $\theta_{c0} = 39.23^\circ$  for the selections.

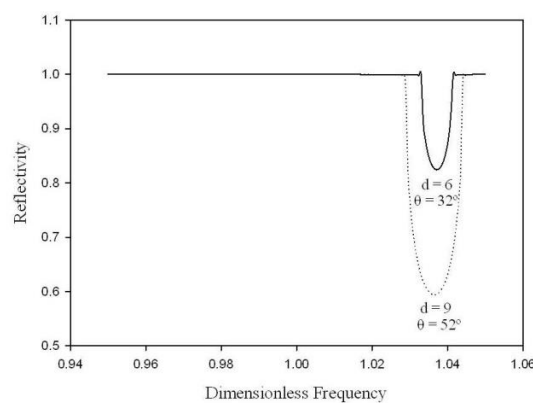
### 3. Results and Discussion

**Fig. 3** shows the superlattice reflectance response with  $d=9$ , and under radiation with different incident angles. As can be seen from different angles, the minimum reflection coefficient depends on frequency and the incident angle. As the incident angle increases, the frequency of the minima increases, but the depth of these minima does not change much. These angles determine the limit conditions for the wave vector to be real or imaginary and thus the phase velocity to be positive or negative [8].



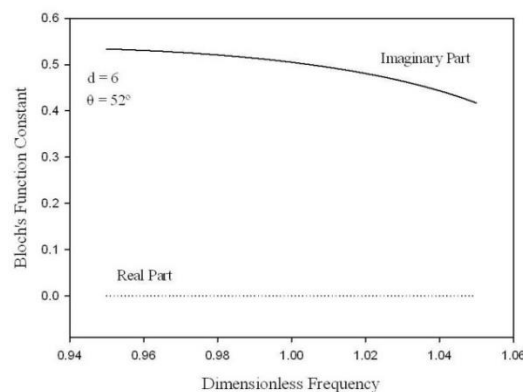
**Fig. 3.** Superlattice response to different radiation incident angles.

**Fig. 4** shows the response of a superlattice with different thicknesses. It is clear from the figure that in this case, the minimum reflection coefficient occurs at different angles and frequencies. One can see that the minima are deeper as the layer thicker. For thicker lattice, the incident angle for minimum reflectivity is more than for thinner lattice. On the other hand, as can be seen, the frequency corresponding to the minimum of the thinner lattice is higher than this frequency for the thicker lattice.



**Fig. 4.** Superlattice response of radiation with different thicknesses.

**Fig. 5** shows the real and imaginary parts of the one-dimensional Bloch wave vector for values of a certain angle of incident ( $\theta = 52^\circ$ ) and the thickness ( $d = 6$ ). As can be seen, the trend of changes in the imaginary part is in terms of decreasing frequency and the real part has a fixed amount.



**Fig. 5.** Real and imaginary parts of a one-dimensional Bloch wave vector.



As observed, this structure could form a left-handed environment, and this phenomenon is predicted to alter the Doppler effect, Cherenkov radiation, and Snell's law. On the other hand, the optical effects in left-handed materials can be adapted to ordinary environments with the computational method presented in this work. The presented results allow tuning the group velocity in left-handed media. The dispersion of the group velocity leads to a broadening of the pulse, which can be compensated by the nonlinearity of the medium.

## 4. Conclusion

In this study, the electromagnetic response of a non-magnetic anisotropic superlattice was investigated using a transfer-matrix formulation combined with Bloch theory. The results demonstrate that by controlling structural parameters such as layer thickness and incident angle, negative phase velocity propagation can be achieved in a periodic superlattice without requiring magnetic activity. This behavior leads to unconventional regimes in which phase propagation and electromagnetic energy transport are decoupled.

From an energy sciences perspective, the ability to engineer and tune electromagnetic energy flow at infrared frequencies represents a key outcome of this work. The proposed superlattice enables controllable reflectivity minima and tailored Bloch wave dispersion, providing direct mechanisms for regulating energy transmission, confinement, and routing within photonic structures. Such tunability is particularly relevant for energy harvesting systems, where efficient coupling and guiding of infrared radiation are critical, as well as for infrared energy management and thermal–photonic control technologies.

Furthermore, the demonstrated control over energy propagation suggests that anisotropic superlattices can function as compact and energy-efficient photonic waveguides and routing elements. These features make the proposed structure a promising candidate for integration into advanced optical architectures aimed at energy processing, distribution, and management. The present results provide a foundation for future investigations into optimized superlattice designs, nonlinear effects, and practical implementations of negative-phase-velocity media in energy-related photonic devices.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No new data were generated in this study. The analysis is based exclusively on publicly available reports and literature sources.

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